A study of the interaction of radio waves

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(*Communicated by Sir Edward Appleton, F.R.S.—Received* 6 *November* 1947)

Experiments are described in which the phenomenon of wave-interaction ('Luxembourg effect') is used to provide information about the height at which radio waves of different frequencies are absorbed in the ionosphere. It is first demonstrated by two crucial experiments that the absorption mechanism suggested by Bailey & Martyn (1934 *a* and *b*) is the true one.

Measurements of the phase of the modulation transferred from one wave to the other by the non-linear absorption process in the ionosphere are described; and it is shown how, by measuring this phase at different modulation frequencies, it is possible to locate the region where the interaction occurs. The results of a series of experiments summarized in tables 2, 3*a* and 3*b* and figures 8 and 9 are discussed.

The conclusion is reached that the frequency with which electrons collide with neutral molecules at a height of about 85 km. is of the order 5×10^5 sec.$^{-1}$, and that this is the height near which the main absorption of waves of frequency 1 Mcyc./sec. and 200 kcyc./sec. are absorbed at night. Waves of frequency 90 and 68 kcyc./sec. are absorbed, and possibly also reflected, below this level. With the approach of dawn the regions responsible for absorbing 1 Mcyc./sec. and 200 kcyc./sec. waves drift apart.

The theory of Bailey & Martyn (1934*b*) and Bailey (1937*a*) is related to modern theories of ionospheric absorption and is restated with the standard nomenclature of Appleton's magneto-ionic theory.

1. Introduction

It is well known that a radio wave travelling through the ionosphere may, under certain circumstances, interact with a second wave in such a way that a modulation imposed on one of them becomes transferred to the other. This effect has been called 'wave-interaction' or, sometimes, the 'Luxembourg effect' because the first observations of the effect were made with one set of waves sent out from the broadcasting station at Luxembourg. In those examples of this phenomenon which were first observed the interaction produced unwanted modulation superimposed on the modulation of the radio station which it was desired to receive. It was therefore natural to call the wave whose reception was desired the *wanted wave* and that from which the undesired modulation was transferred was called the *interacting wave*. We shall preserve this nomenclature throughout this paper: it is illustrated schematically in figure 1.

Many experiments have been made to investigate wave-interaction, and especially to discover the factors which determine its magnitude (van der Pol & van der Mark (1935); van der Pol (1935); Baümler & Pfitzer (1935); Grosskopf (1938); and Huxley, Foster & Newton (1947)). Interaction has so far been observed only with wave-lengths in the broadcast band (200 to 1500 m.) where it has been found with wanted and interacting stations both in the medium-wave band, both in the long-wave

band, or with one in each band. In the latter case the wanted wave may be either in the medium-wave or the long-wave band.

To produce interaction, the ionosphere must behave as a 'non-linear' transmission medium, and Bailey & Martyn (1934*a*) have suggested that the absorption of the waves provides the necessary non-linearity. They first put forward their suggestion before ideas about the ionospheric absorption of waves were very clear; their theory may now be restated as follows. The main absorption of the wanted wave occurs over fairly well-defined portions of the trajectory which are at a level different from that where the main bending takes place. This absorption depends on the frequency with which electrons collide with neutral molecules, and anything which increases the velocity of the electrons, and therefore the frequency of these collisions, will increase the absorption. If now another wave (the interacting wave) is absorbed in the same place the energy removed from it will increase the velocity

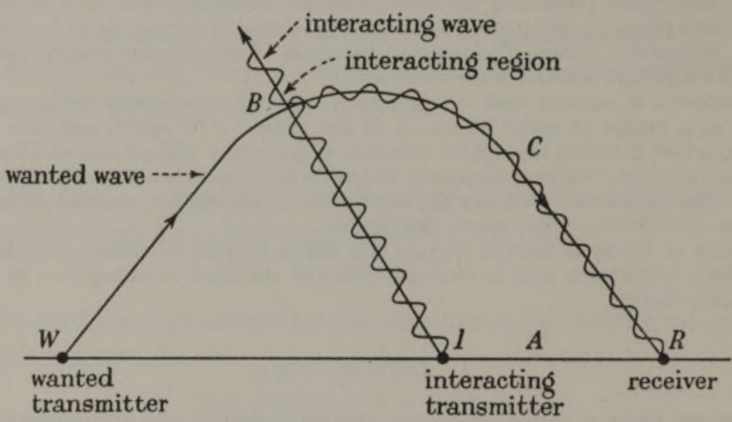

FIGURE 1. Schematic diagram to illustrate nomenclature.

of the electrons,* and therefore the frequency of their collisions, with the result that the wanted wave will be more strongly absorbed. In the presence of the interacting wave the wanted wave will therefore be weaker. If the amplitude of the interacting wave is now slowly varied periodically the velocity of the electrons will follow in step, and the absorption of the wanted wave will also vary periodically: in this way the modulation of the interacting wave will become superimposed on the wanted wave. According to the theory of Bailey & Martyn this is the mechanism of wave interaction.

If the interacting wave were suddenly removed the velocities of the electrons would fall exponentially to their normal values with a time-constant which would depend on the energy lost by an electron at each collision. We shall call this time-constant the 'relaxation time' of the electrons and shall denote it by τ; we shall

* It is interesting to note that a 100 kW transmitter working on a frequency of 200 kcyc./sec. is capable of increasing the temperature of the electrons in the appropriate part of the ionosphere by about 2% (see § 6 (*a*)).

see later that its magnitude is of the order of 10^{-3} sec. If the modulation frequency is increased so that it becomes comparable with $(2\pi\tau)^{-1}$ then the electrons will begin to be incapable of following the modulation and the interaction will become smaller. For slow variations the electrons' velocity will vary in phase with the modulation of the interacting wave; as the frequency is increased, however, the electrons' velocity will tend to lag in phase behind the modulation of the wave, and when the frequency is great the phase-lag will be $\frac{1}{2}\pi$.*

The original papers of Bailey & Martyn contained the deductions mentioned above, but the theory is here restated in detail in § 6 and appendix A in a form which relates it more closely to modern theories of the ionosphere. If the theory is correct it might be expected that an experimental study of wave interaction would yield valuable information about the absorbing portions of the ionosphere, and in particular about those parts of the ionosphere which absorb both waves strongly. It is the purpose of this paper to describe experiments made with this object, and to discuss the results obtained.

Before the present work was started the only experimental test of the theory of Bailey & Martyn was one in which the amplitude of the transferred modulation was measured at different frequencies (van der Pol & van der Mark 1935). It was found that the shape of the curve relating amplitude and frequency agreed with the theory but the test was admittedly somewhat indirect, and van der Pol stated (van der Pol 1934):

'Finally in our opinion it cannot be considered to have been proved definitely that the effect is of a typical non-linear nature. The possibility is still open for an explanation on the basis of a Doppler effect, viz. the periodic raising of the conducting layers of the ionosphere in the rhythm of the modulation of the unwanted station and restored by the molecular agitation, thus causing a frequency or phase modulation of the wanted wave, which may in turn cause an amplitude modulation at the receiver due to the superposition of this frequency modulated wave with other rays which have not been affected on their course.'

It therefore seemed desirable to make a more direct test of the absorption hypothesis, and two such tests are described in § 3. The first of these consisted in switching the interacting wave on and off and observing that the wanted wave was weaker in the presence of the interacting wave. The second was suggested by Appleton (1938) and involved a comparison, at low modulation-frequencies, of the phases of the modulation received direct on the interacting wave and received after being transferred to the wanted wave by the process of wave-interaction. Both these tests supported the hypothesis of the absorption mechanism.

Once interaction had been demonstrated by switching the interacting wave on and off it was no longer necessary to modulate that wave to make observations, and it was therefore possible to use wave-lengths outside the broadcast band, where the transmitting stations available are not usually capable of being modulated. Experiments on these wave-lengths are described in § 5.

* For a proof of these statements see appendix A.

When a modulated interacting wave was used interesting information was obtained by varying the modulation frequency and comparing the phase of the modulation received direct from the interacting station with that of the modulation transferred to the wanted wave. As the frequency is changed the phase difference changes for two different reasons: (*a*) the phase change associated with the relaxation time occurs as previously described, and as analyzed in detail in appendix A; and (*b*) a progressive phase change occurs associated with the different lengths of the two paths labelled *IAR* and *IBCR* in figure 1. It has proved possible to separate the two different effects and to deduce separately the relaxation time of the electrons and the difference between the paths *IBCR* and *IAR*. The path *WBCR*, of the wanted wave, was determined, in a separate experiment, by causing the transmitter of the wanted wave to emit pulses of the type usual in ionospheric research and it was then possible to locate the region (*B*) of interaction. Experiments based on the principles here outlined are described in § 4.

In the estimation of the relaxation time of the electrons it is also of value to observe the way in which the amplitude of the imposed modulation varies with frequency. A special method developed for measuring this amplitude is described in § 5.

The evidence from the different types of experiment is collected and discussed in § 7. In this discussion we shall make use of a quantity which we have called the 'coefficient of transferred absorption' which is a measure of the fractional decrease which occurs in the field strength of the wanted wave when a steady interacting wave is suddenly switched on.

2. Arrangement of transmitters and receivers

For experiments of this kind it is necessary to use transmitters of suitable wave-length and considerable power situated in suitable places, and we were most fortunate in obtaining the enthusiastic co-operation of the B.B.C., who provided the necessary transmissions. We wish to record our thanks to the Chief Engineer and all those on his staff who helped in the organization and carrying out of the complicated series of experimental transmissions for which we asked and, in particular, we should like to thank Mr H. L. Kirke, head of the B.B.C. Research Department, for his willingness to help on all occasions. We also wish to thank the Chief Engineer of the Post Office and his staff, and the Director of Scientific Research at the Admiralty, for putting their most valuable facilities at our disposal. Without the help of these organizations the series of experiments here described would have been impossible.

The main series of observations was made at Cambridge (lat. 00° 05′ W., long. 52° 10′ N.). Dr L. G. H. Huxley made some additional observations on the same transmissions at Birmingham and we have had the benefit of comparing his results with ours. The transmitters which were used at different times are listed in table 1 and their positions indicated in the map of figure 2.

TABLE 1. DETAILS OF TRANSMITTERS USED

name or call sign	place	lat. and long.	designation on map of figure 2	controlling authority	power in kW	frequency in kcyc./sec.	nature of transmission
Ottringham	Ottringham	00° 04′ W. 53° 42′ N.	O	B.B.C.	167	167	modulated
Droitwich	Droitwich	02° 06′ W. 52° 18′ N.	D(200)	B.B.C.	150	200	modulated
Westerglen	Westerglen	03° 49′ W. 55° 59′ N.	W	B.B.C.	60	767	modulated
Lisnagarvey	Lisburn	06° 03′ W. 54° 29′ N.	L	B.B.C.	100	1050	modulated
Droitwich	Droitwich	02° 06′ W. 52° 18′ N.	D(1013)	B.B.C.	68	1013	modulated
GBY	Rugby	01° 20′ W. 52° 15′ N.	GBY	Post Office	80	60	single sideband telephony
GYA 2	Cleethorpes	00° 10′ W. 53° 30′ N.	GYA 2	Admiralty	20	90·2	keyed

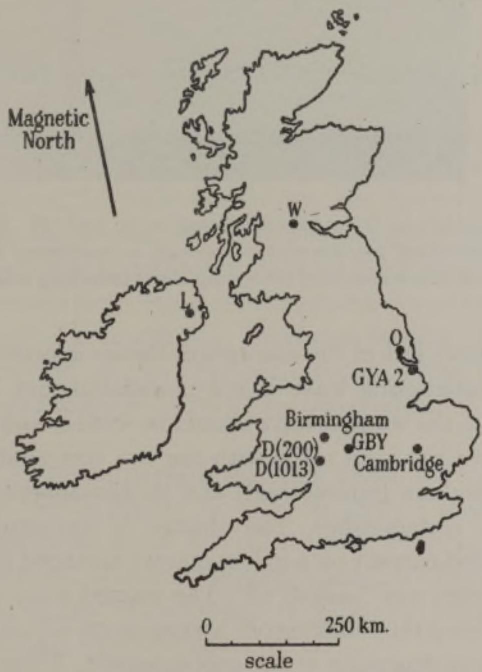

FIGURE 2. Locations of transmitters and receivers.

3. Tests of the absorbing mechanism

If wave interaction is explained by the absorption theory of Bailey & Martyn, then when the interacting wave is strong we should expect the wanted wave to be weak, and Appleton (1938) has pointed out that the modulation of the interacting wave and the modulation imposed on the wanted wave should therefore be in antiphase. It is shown in the next section that this antiphase relationship would only be expected if the modulation frequency were low enough, and it is demonstrated that a frequency of 50 cyc./sec. satisfies the necessary conditions. Special attention has therefore been paid to the observation of the phases at this frequency. On all occasions, and with all the combinations of wanted and interacting transmitters which are described later, it has been found that the modulation received direct from the interacting station is in opposite phase to that transferred to the downcoming wave from the wanted station.* This observation provides one fundamental check of the absorption theory of wave-interaction.

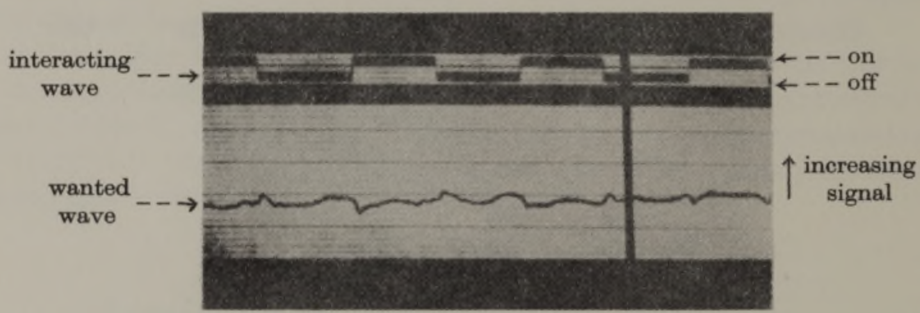

Figure 3. Effect of switching the interacting station on and off. In this record the wanted signal is fed to the recording galvanometer through a condenser so that only the sudden changes of amplitude, at the instants of switching the interacting wave on or off, are recorded.

A further very direct test of the absorption theory is provided by arranging that the sender of the interacting wave is not modulated, but is simply switched on and off. When it is on the wanted wave should be weaker and when it is off it should be stronger. This test has been made with the two combinations of (*a*) Westerglen (wanted) and Ottringham (interacting); and (*b*) Lisnagarvey (wanted) and Droitwich 200 kcyc./sec. (interacting). The change of amplitude was measured by observing the rectified current on a galvanometer arranged so that the greater part of the standing current was 'backed off'. The wanted wave was always fading and it was necessary to keep the 'backing off' arrangement adjusted by hand so that the reading was always on the scale of the galvanometer. There was a clearly marked decrease of amplitude when the interacting station was switched on and an increase when it was switched off, as required by the theory.

* In appendix B we discuss some complications which may arise if a ground wave is received in strength comparable with the downcoming wave.

The change in amplitude ΔF of the radio-frequency e.m.f. was determined as a fraction of the total e.m.f. F. We shall call the fraction $\Delta F/F$ the 'coefficient of transferred absorption', and its magnitude will be further discussed in § 5.

Although the method of the 'backed-off' galvanometer proved most satisfactory for measurement of the change of amplitude, the occurrence of fading rendered it less useful for providing a permanent record of the effect. A photographic record was therefore obtained by the use of a rapidly moving galvanometer which was fed from the receiver through a condenser of such a value that the slower changes representing fading were less evident in comparison with the rapid changes produced by keying the interacting transmitter. A record obtained in this way is reproduced in figure 3.

The success of these two very direct tests of the absorption theory of interaction has satisfied us that the theory is correct, and has encouraged us to use it in making the detailed deductions explained in the rest of this paper.

4. Observations of phase

Since it was desired to compare the phase of the transferred modulation with that of the wave received over a known direct path from the interacting station, it was essential to ensure that the downcoming wave alone was received from the wanted station and the ground wave alone from the interacting station. In the two main series of experiments to be described in this section the transmitter of the wanted wave was either Westerglen (frequency 767 kcyc./sec., distance 480 km.) or Lisnagarvey (frequency 1050 kcyc./sec., distance 495 km.) and, since the experiments were all done during the night or at dawn, the downcoming wave was considerably stronger than the ground wave. It was therefore sufficient to receive it on a simple aerial; a loop was often used so that its directional properties could be used to reduce interference from other transmissions. In some subsidiary experiments the wanted station was nearer (e.g. Droitwich distant 180 km., transmitting either on 1013 kcyc./sec. or on 200 kcyc./sec.) and in order to receive only the downcoming wave it was necessary to use a loop aerial adjusted, by day, to suppress the ground-wave.

In the main series of experiments the interacting transmitter was in the long-wave band and was distant 100 or 200 km. so that its ground wave was considerably stronger than the downcoming wave. A vertical aerial, which discriminates to some extent against the downcoming wave, was always used. No deductions about phase were made from any subsidiary experiments in which the interacting station produced a strong downcoming wave at the receiver.

The two waves were received on two separate radio-frequency amplifiers (three stages of straight amplification) each followed by a diode detector and one stage of audio-frequency amplification. The audio-frequency amplifiers were designed so that their response was constant over the range 50 to 2000 cyc./sec. and so that the phase-shift introduced by them was small. Moreover, both amplifiers were similar

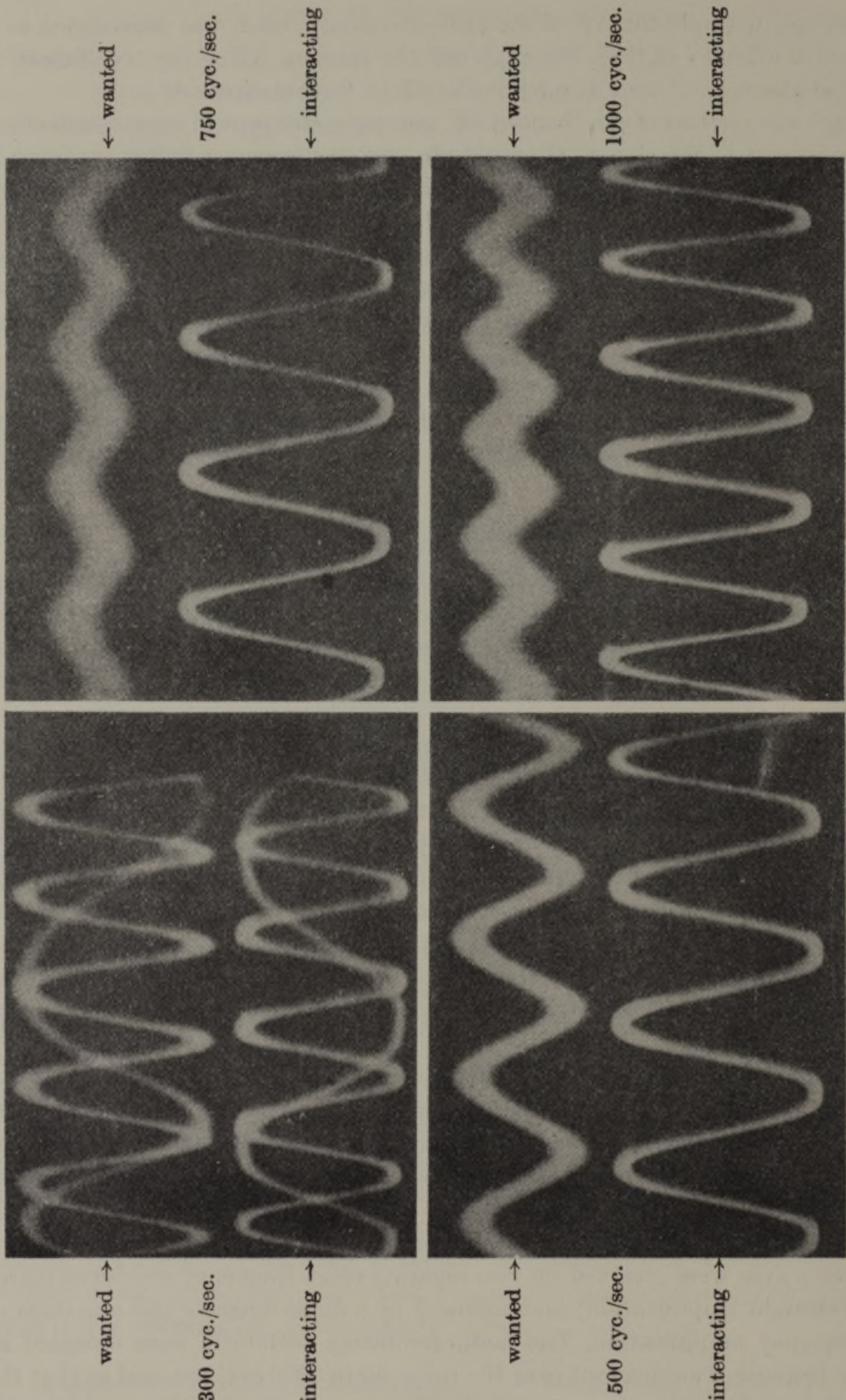

FIGURE 4. Records showing phase relation between the modulation received direct by the interacting wave, and indirectly after being transferred to the wanted wave by the process of wave-interaction. There is a progressive shift of phase as the modulation frequency is increased.

so that any residual phase-shift would be the same in each. The outputs of the two amplifiers were applied to deflect the two traces of a Cossor double-beam cathode-ray oscilloscope and were viewed on a linear time-base synchronized to the modulation. The equipment was tested by the application of radio-frequency e.m.f.'s generated in local signal-generators, modulated from a common source, and the amplitude of the wave-trace on the oscilloscope was shown to be proportional to the modulation amplitude of the input and independent of the frequency. The two traces were shown to be in phase, and to remain in phase over the operational frequency range. Tests were also made to demonstrate that no interaction occurred between the two amplifiers, and possible interaction in the oscilloscope was guarded against.

The method of experimentation usually adopted was to arrange for the wanted wave to be unmodulated, and for the interacting wave, usually with a power of 100 or 150 kW, to be modulated to a depth of 80 % with a series of pure tones having the frequencies 50, 75, 100, 150, 200, 300, 400, 500, 750, 1000, 1250, 1500, 1750, 2000 cyc./sec., the whole sequence of operations occupying about half an hour. The traces on the oscilloscope were either observed visually, or photographed and afterwards measured up, and the phase difference at each frequency was determined. A good measure of the phase difference could usually be obtained on all frequencies less than about 1000 cyc./sec. Examples of the type of trace observed are shown in figure 4.

When the observed phase-difference was plotted against modulation frequency a curve of the type shown in figure 5 was obtained. This curve may be explained in terms of the ideas outlined in § 1 in the following way. We show in appendix A that, as a result of the finite 'relaxation time' τ of the ionospheric electrons, the transferred modulation at a frequency f has a phase-lag $\phi_1 = \tan^{-1}(2\pi f\tau)$. If, also, there is a difference d between the two paths IAR and $IBCR$ of figure 1, there is an additional phase-lag given by $\phi_2 = 2\pi fd/c$. The total phase-lag ϕ is then the sum of the 'electron phase-lag' ϕ_1 and the 'path phase-lag' ϕ_2, so that

$$\phi = \tan^{-1}(2\pi f\tau) + 2\pi fd/c.$$

When a set of experimental points had been plotted as in figure 5 the constants τ and d were chosen so as to give a theoretical curve lying smoothly among the points. The curve in figure 5 is drawn for $\tau = 1{\cdot}8 \times 10^{-3}$ sec. and $d = 222$ km. In figure 6 the 'path phase-lag' ϕ_2 appropriate to this distance has been subtracted from the experimentally observed phase-lags and the resulting 'observed' values of ϕ_1 plotted as points. The curve represents $\phi_1 = \tan^{-1}(2\pi f\tau)$ for $\tau = 1{\cdot}8 \times 10^{-3}$ sec. and the accuracy with which the points fall on it provide a measure of the reliability of the deductions.

The composition of the downcoming wave was frequently investigated, during a night's observations, by arranging that pulses of duration about 100 μsec. were emitted from the wanted station and observed at the receiver on an oscilloscope with a time-base synchronized, by way of the national 'grid' system of electric supply, to the emission of the pulses. A measurement of the intervals between the

pulses sufficed to decide whether a given pulse was reflected from region E or region F but it was not usually possible to make measurements with sufficient accuracy to provide a reliable measure of the height of the reflexion; this is particularly the case because the difference of path between the different multiple reflexions arriving at oblique incidence is not very sensitive to variations of the height of reflexion. We have therefore usually assumed that reflexions from region E occurred at a height of 100 km. and those from region F at 250 km.; these represent average heights of

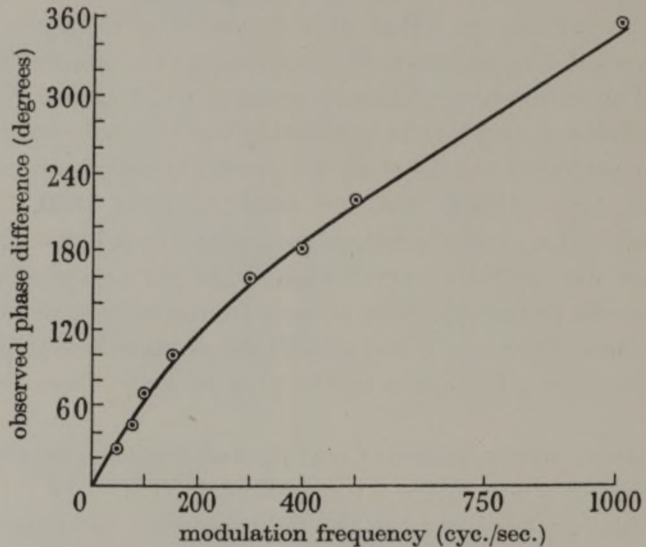

FIGURE 5. Experimental results obtained at 22.30 to 02.30 hr. G.M.T. on 4/5 May 1947, with Westerglen as wanted station and Ottringham as interacting station.

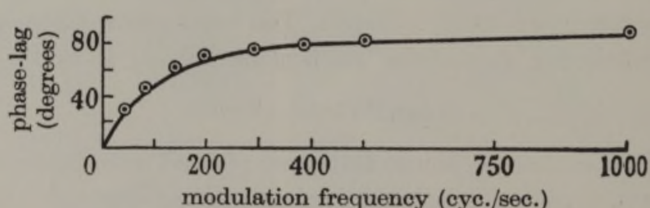

FIGURE 6. Curve of electron phase-lag against frequency. Experimental points marked ⊙ are deduced from figure 5; the full line is the curve $\phi = \tan^{-1}(2\pi f \times 1{\cdot}8 \times 10^{-3})$.

reflexion for waves of this frequency incident obliquely, as deduced from the routine ionospheric observations carried out at the Radio Research Station, Slough (lat. 00° 45′ W., long. 51° 25′ N.).* Our measurements of echo-patterns have fitted these values quite well. Comparison of successive reflexions from the same region also provided a measure of the absorption; this information will be used in the discussion contained in § 7.

* We are much indebted to Dr R. L. Smith-Rose for supplying us with detailed information for the occasions of our experiments.

Some observations of the pulse transmissions have been made in the presence of a modulated interacting wave and the modulation transferred to one single reflected pulse, generally the first from region E, has been observed. It is probable that useful information could be derived from further observations made in this way.

In the theoretical interpretation of the curves of figures 5 and 6 we have assumed that there is one predominant downcoming wave from the wanted station and that all the interaction occurs at one point on its trajectory. The more complicated situation arising if these assumptions do not hold is analyzed in appendix B. On some occasions we have noticed that the phase of the transferred modulation alters during the 2 min. period of the observation; this is explained in the appendix as being due to the presence of two downcoming waves comparable in intensity. In general, however, the phase was constant during a period of observation, as we should be led to expect from the pulse transmissions which usually showed the first echo to be much stronger than the others.

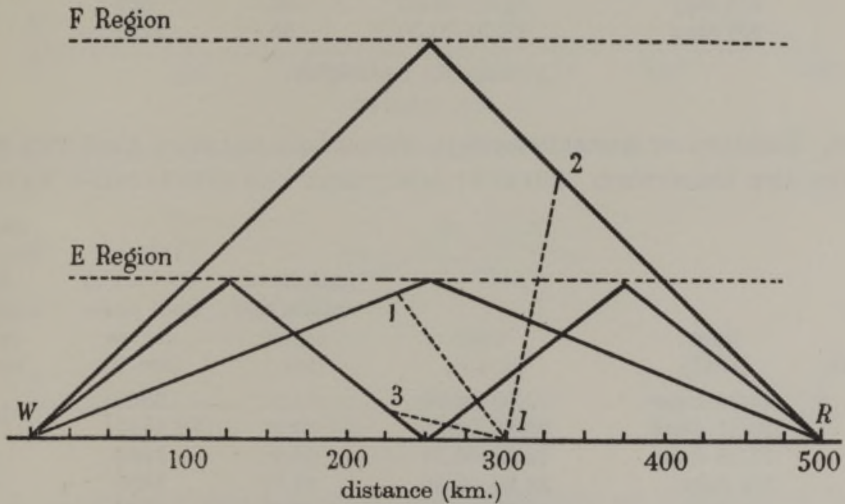

FIGURE 7. Possible locations of wave-interaction deduced from experiments, summarized in figures 5 and 6, which give an average path difference of 200 km.

From curves such as that of figure 5 the path difference d was deduced, and since the distance IAR in figure 1 was known, the distance $IBCR$ was calculated. Let us call this distance x. A series of possible wave-trajectories from the wanted station to the receiver was then drawn, as indicated in figure 7, and points (such as 1, 2, 3) on these trajectories were found which gave the measured value of x. The points marked in figure 7 correspond to the experimental results summarized in figure 5, in which Westerglen was used as the wanted station and Ottringham as the interacting station. In choosing amongst these possible positions for the origin of the wave-interaction we are now guided by the expectation that the interaction will be greatest where there is the greatest amount of absorption common to the two waves, and this region is just below the point of reflexion in region E. In the case illustrated in figure 7 we therefore select the point 1 as the probable location of the interaction.

Two series of experiments of the type outlined above have been carried out with the object of locating the region where the interaction occurs. In the first series the wanted wave was trasmitted from Westerglen and the interacting wave from Ottringham. The results are summarized in table 2 and figure 8. In the second series the wanted wave was transmitted from Lisnagarvey and the interacting wave from

TABLE 2. RESULTS OF EXPERIMENTS IN WHICH WESTERGLEN SENT THE WANTED WAVE AND OTTRINGHAM SENT THE INTERACTING WAVE

number of experiment	date (1947)	time (G.M.T.)	height of interaction region (km.)	value of $G\nu$ $(=1/\tau)$ from phase curves (sec.$^{-1}$)	value of $G\nu$ $(=1/\tau)$ from amplitude curves (sec.$^{-1}$)
1	10 January	01.00–02.30	95	623	725*
2	11 January	01.00–02.30	95	895	
3	4/5 May	22.30–00.30	90	555	—
4	5/6 May	22.30–00.30	85	1260	—

* Average for two nights.

TABLE 3a. RESULTS OF EXPERIMENTS IN WHICH LISNAGARVEY SENT THE WANTED WAVE AND DROITWICH (200 KCYC./SEC.) SENT THE INTERACTING WAVE

number of experiment	date (1947)	time (G.M.T.)	height of interaction region (km.)	value of $G\nu$ $(=1/\tau)$ from phase curves (sec.$^{-1}$)	value of $G\nu$ $(=1/\tau)$ from amplitude curves (sec.$^{-1}$)
5	15/16 June	22.30–02.30	77·5	2520	1640†
6	16/17 June	22.30–02.30	83·5	1640	
7	17/18 June	22.30–02.30	75·0	1260	—
8	3/4 July	22.30–02.30	91·5	1320	—
9	4/5 July	22.30–02.30	97·0	2890	—
10	5/6 July	22.30–02.30	78·0	1290	—
11	6/7 July	22.30–02.30	87·0	1010	—
12	6 August	23.00–23.30	84·0	890	—
13	7 August	00.10–00.30	80·0	720	—

† Average for three nights.

TABLE 3b. RESULTS OF EXPERIMENTS IN WHICH LISNAGARVEY SENT THE WANTED WAVE AND OTTRINGHAM THE INTERACTING WAVE

number of experiment	date (1947)	time (G.M.T.)	height of interaction region (km.)	value of $G\nu$ $(=1/\tau)$ from phase curves (sec.$^{-1}$)	value of $G\nu$ $(=1/\tau)$ from amplitude curves (sec.$^{-1}$)
14	15/16 June	22.30–02.30	85	—	—
15	16/17 June	22.30–02.30	92	1570	—

Droitwich or Ottringham. The results are summarized in table 3 and figure 9. In tables 2 and 3 the results of experiments are expressed in terms of the quantity $1/\tau$, since it is shown, in equation (21) of appendix A, that this is equal to the theoretically important quantity $G\nu$.

In the first series of experiments the interacting station was nearly on the straight line joining the receiver and the wanted station, in the second series the interacting station was some considerable distance from this line and the appropriate solid geometry had to be taken into account in calculating the location of the interaction.

The results of the phase measurements, as summarized in tables 2 and 3 and figures 8 and 9, are discussed in § 7.

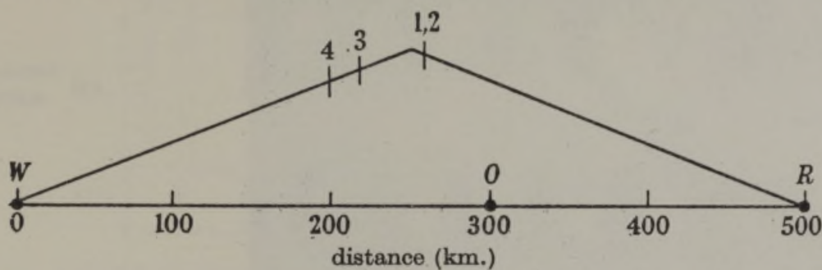

FIGURE 8. Locations of observed interaction of Ottringham on Westerglen (see table 2).

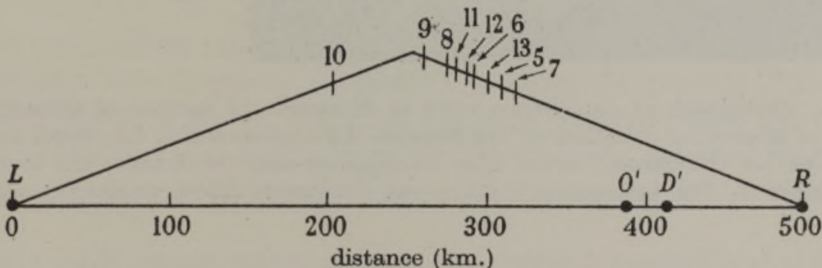

FIGURE 9. Locations of observed interaction of Ottringham and Droitwich on Lisnagarvey (see tables 3*a* and 3*b*). The points O' and D' represent the feet of the perpendiculars from Ottringham and Droitwich to the line joining Lisnagarvey and Cambridge.

5. MAGNITUDE OF THE INTERACTION

In order to measure the relative magnitude of the transferred modulation, a simple method was devised which gave good results even with a wave which exhibited marked fading. It was arranged that the wanted wave was weakly modulated with an amplitude of 5 or 10 % at a frequency somewhat different from the modulation frequency of the interacting station, so that, at the receiver, the normal modulation of the station was observable in addition to the modulation imposed by interaction. The trace on the oscilloscope then took the form shown in figure 10, in which the distance AB corresponds to the directly-impressed modulation and the distance CD to the transferred modulation. From oscillograms of this kind it was a simple matter to deduce the percentage modulation due to interaction. Although the

percentage of direct modulation as observed at the receiver changed, due to interference effects between different reflected waves, and although the amount of interaction also varied as the wave faded, it was found that the mean readings taken in this way were fairly constant and appeared to be reliable. They also agreed with direct measurements made by recording simultaneously the rectified current, which gave a measure of the carrier intensity, and the amplitude of the oscilloscope trace, and then calibrating with a modulated e.m.f. from a signal generator. Measurements made in this way led to the results described in the following sub-sections.

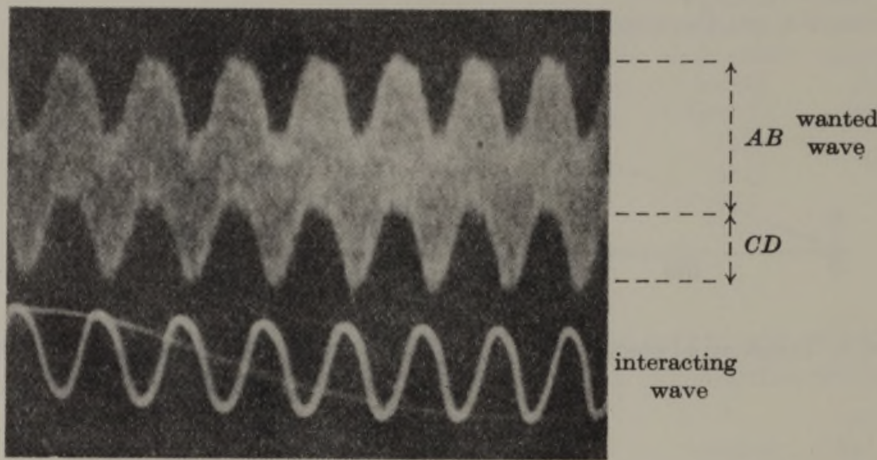

FIGURE 10. Photograph of oscilloscope trace to illustrate the method of measuring small amplitudes of interaction-modulation. The distance AB corresponds to 5 % direct modulation at 500 cyc./sec. on the wanted carrier from Lisnagarvey and the distance CD to the transferred modulation. The interacting station was Droitwich (200 kcyc./sec.) modulated at 100 cyc./sec.

(a) *Magnitude of interaction as a function of modulation frequency*

The results of a typical series of measurements made as the modulation frequency was varied are shown in figure 11 and are similar to those obtained previously by other workers (van der Pol & van der Mark 1935; Huxley *et al.* 1947).

Equation (20a) of appendix A shows that we should expect the coefficient of modulation (m) superimposed on the wanted wave to be given by the expression

$$m = T\{2\mu/(1+\tfrac{1}{2}\mu^2)\}\{1+4\pi^2f^2\tau^2\}^{-\frac{1}{2}}. \tag{20a}$$

In this expression T is the 'coefficient of transferred absorption' defined by equation (8) of § 6; μ is the coefficient of modulation of the interacting wave modulated at frequency f; and τ is the relaxation time of the ionospheric electrons.

That the experimental points plotted in figure 11 show a reasonable agreement with theory is seen by comparing them with the theoretical curve which has been plotted for $\tau = 5{\cdot}8\times10^{-4}$ sec. On all occasions when the intensity of the interaction has been determined as a function of frequency in this way the agreement with

theory has been similar to that shown in figure 11 and the values obtained for the relaxation time τ are shown in tables 2 and 3. These values are discussed in §7. An examination of all our results shows that the agreement with theory is not so good in the case of amplitude measurements as it is in the case of phase measurements. This difference, which may be seen by comparing figure 11 with figures 5 and 6, is difficult to explain.

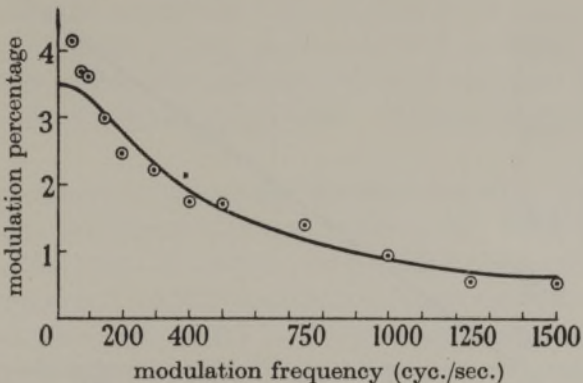

FIGURE 11. Percentage transferred modulation as a function of modulating frequency. Results obtained on 16/17 June, with Lisnagarvey sending the wanted wave and Droitwich the interacting wave. Experimental points ⊙; full curve theoretical values for $\tau = 5{\cdot}8 \times 10^{-4}$ sec.

If, in a plot such as that of figure 11, the curve showing best theoretical fit is extrapolated to zero frequency we obtain a value for the quantity $T\{2\mu/(1+\frac{1}{2}\mu^2)\}$ and hence, since μ is known, for T. It has been confirmed that the value of T obtained in this way is in good agreement with the value obtained from measurements in which the unmodulated interacting carrier was switched on and off (see §3).

We have made enough measurements of modulation amplitude as a function of modulation frequency to satisfy ourselves that the results are approximately represented by equation (20a) and we shall usually express the absolute magnitude of the interaction effect by stating the value of the 'coefficient of transferred absorption', T, deduced from curves of the type shown in figure 11, or measured directly by switching the interacting station on and off.

(b) *Coefficient of transferred absorption as a function of the power of the interacting station*

The theory of Bailey & Martyn (1934b) would lead to the expectation that the 'coefficient of transferred absorption' should be proportional to the radiated power of the interacting transmitter, but some tests by Grosskopf (1938) did not, apparently, confirm this expectation. We therefore conducted a special experiment in which the interacting wave was emitted from Ottringham and was modulated at a constant frequency of 400 cyc./sec. and the modulation impressed on a wanted wave emitted by Westerglen was measured, by the method described above, for different powers

of the interacting transmitter. The results are shown in figure 12 and appear to agree well with the theoretical expectation, in contrast with the results of Grosskopf. We have little doubt that the facts are in agreement with theory in this respect, but cannot suggest why the other workers found a different result.

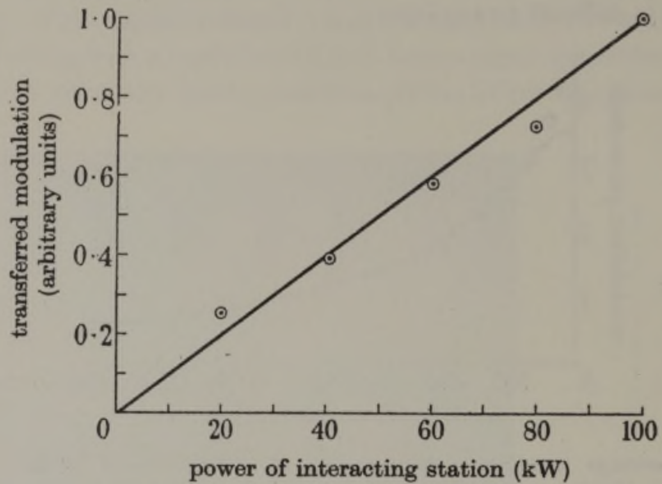

FIGURE 12. Results of varying the power of the interacting station, obtained at 00.30 hr. G.M.T. on 5 to 9 May, with Westerglen sending the wanted wave and Ottringham the interacting.

TABLE 4. OBSERVED VALUES OF COEFFICIENTS OF TRANSFERRED ABSORPTION

	sender of wanted wave			sender of interacting wave			coefficient of transferred absorption		
serial number	symbol on figure 2	frequency (kcyc./sec.)	power (kW)	symbol on figure 2	frequency (kcyc./sec.)	power (kW)	observed	reduced to 100 kW, 150 km.	method of observation
1	W	767	60	O	167	167	0·08	0·05	modulation and keying
2	W	767	60	GYA 2	90·2	20	<0·001	<0·005	keying
3	L	1050	100	D(200)	200	170	0·04	0·023	modulation
4	L	1050	100	D(1013)	1013	60	0·03	0·029	modulation
5	L	1050	100	GBY	68	80	<0·001	<0·002	keying and modulation
6	L	1050	100	O	167	167	0·01	0·011	modulation
7	D(200)	200	170	D(1013)	1013	60	0·02	0·024	modulation*
8	D(1013)	1013	60	D(200)	200	170	0·04	0·029	modulation*
9	D(200)	200	170	O	167	167	0·04	0·029	modulation*
10	O	167	167	(D200)	200	170	0·02	0·014	modulation*

* Loop: ground ray suppressed.

(c) *Coefficient of transferred absorption with different pairs of stations*

An attempt was made to measure the 'coefficient of transferred absorption' with different pairs of sending stations to investigate in detail the effect of the radio frequencies and geographical positions of the senders. The powers of the interacting stations were not the same, and in order to reduce the results to a common basis for comparison the measured coefficients were expressed, by using the results of sub-section (*b*), in terms of an interacting transmitter of power 100 kW, distant 150 km. from the point of interaction. We shall refer to these modified coefficients as 'equivalent coefficients of transferred absorption'. The results obtained are shown in table 4.

Most of the measurements were straightforward and were made by the methods described in previous paragraphs. In one or two cases, however, special arrangements were used; thus when the morse station GYA 2 was used as sender of the interacting wave it was not possible to modulate it and the tests had to be made by keying it on and off. The station GBY is constructed for single sideband telephony and the tests in which it emitted the interacting wave were made in two different ways. In one an audio-frequency tone was injected into the speech circuit and was keyed on and off; this produced an intermittent sideband at one frequency which simulated a keyed C.W. signal. In the other method the audio tone was itself modulated at 50 cyc./sec. so that the single sideband emitted was effectively a single carrier modulated at 50 cyc./sec.

The ultimate sensitivity of observation was set by the random noise received and when no interaction was observed an upper limit is given in table 4. The arrangement could have been made more sensitive by the use of narrow-band audio filters and it is proposed to use this method in future work.

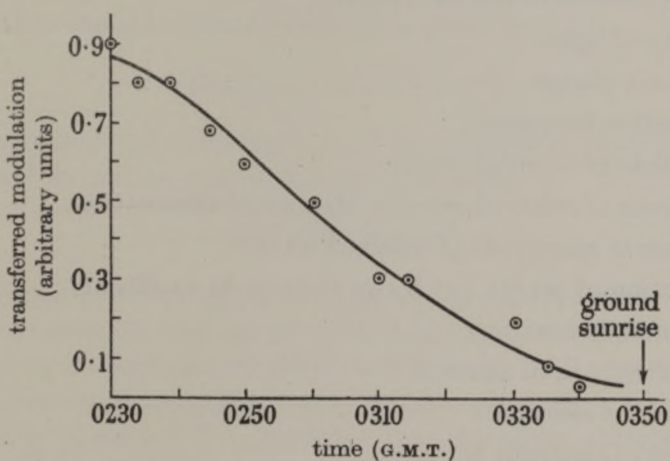

FIGURE 13. Decrease of interaction as dawn advances. Results of 7 July, with Lisnagarvey sending the wanted wave and Droitwich the interacting. Sunrise in the ionosphere at 100 km. was at 02.30 hr. G.M.T.

(*d*) *The effect of sunrise on the coefficient of transferred absorption*

It is well known that the intensity of downcoming waves decreases near sunrise due to increased absorption, and it is of interest to measure the coefficient of transferred absorption throughout this period. Experiments were made with Lisnagarvey as wanted station and Droitwich (200 kcyc./sec.) as interacting station during the sunrise period on 7 July 1947.* The wanted station was modulated to a depth of 10 % (or sometimes 5 %) and the comparison method previously outlined was used. The results are shown in figure 13 from which it can be seen that the coefficient of transferred absorption was greatest during the night and became unmeasurably small during the hour before ground sunrise.

6. Theoretical considerations

We shall now consider, in the light of present-day theories of absorption, the relation between interaction phenomena and the absorption of waves passing through the ionosphere. We shall calculate the magnitude of interaction effects in two parts. In the first part, contained in this section, we shall calculate the interaction effect at zero modulation frequency (corresponding to the switching on and off of the interacting wave), and shall express it in terms of a coefficient of transferred absorption (T), which we have already defined. In the calculations we shall refer to our ideas about the absorption of waves, so that our calculations; although allied to those of Bailey & Martyn, will be expressed somewhat differently. The second part of the calculation concerns the effect of the modulation frequency and is exactly that previously given by Bailey & Martyn. For the sake of completeness it is included, with our terminology, in appendix A.

We shall use the following nomenclature, in which we adhere to that used by Appleton in his theories of the ionosphere:

c = velocity of light.
e = electronic charge.
f = modulation frequency.
F = amplitude of received wave.
F_0 = amplitude of received wave in absence of attenuation.
ΔF = decrease in amplitude of received wave.
G = proportion of energy lost by an electron in a collision.
k = Boltzmann's constant.
l = mean free path of electron.
m = mass of the electron.
N = number of electrons per c.c.
p = angular frequency of the carrier.

* The exigencies of the broadcasting service, in conjunction with the use of Double British Summer Time, rendered it impossible to do these experiments except near midsummer.

$p_L = H_L e/mc$ = angular gyro-frequency of electrons in the longitudinal component of the earth's magnetic field.

$p_T = H_T e/mc$ = angular gyro-frequency of electrons in the transverse component of the earth's magnetic field.

P = power per unit area in the wave.

T = absolute temperature.

u = velocity of the electron.

U = gas-kinetic energy of the electron.

ΔU = increase in gas kinetic energy in presence of wave.

W = work done by the wave on each electron per second.

x = distance along wave normal.

α = a constant given by relation $\alpha\nu = \rho$.

ϵ_0 = permitivity of free space.

κ = absorption coefficient of wave per unit path.

ν = frequency of collision of electrons with molecules.

$\Delta\nu$ = change in collision frequency due to incident field.

ρ = attenuation of wave in 'nepers'.

(*a*) *The magnitude of wave-interaction*

The theory of ionospheric absorption is considerably complicated by the presence of the earth's magnetic field, and for simplicity we first consider what would happen in the absence of that field. Investigations of absorption of radio waves in the ionosphere (Farmer & Ratcliffe 1935) have led to the view that the waves are most strongly absorbed at a level where the refractive index does not depart much from unity, and that then the absorption coefficient is given by:

$$\kappa = \{2\pi N e^2/\epsilon_0 cm\}\{\nu/(p^2+\nu^2)\}. \tag{1}$$

There are also reasons for thinking that all the important absorption occurs at levels where $p^2 \gg \nu^2$ so that we may write approximately

$$\kappa = \{2\pi N e^2/\epsilon_0 cm\}\{\nu/p^2\}. \tag{2}$$

As it travels through the ionosphere the interacting wave is absorbed at each level as given by this expression, and the power lost by it represents a 'heating up' of the ionospheric electrons and an increase in their velocity. If P represents the power per unit area in the wave at any given level in the ionosphere, then it is absorbed near that level according to the law $P = P_0 e^{-2\kappa x}$ so that the power absorbed in a slab of unit cross-section and of thickness dx is given by

$$-\frac{dP}{dx}dx = 2\kappa P\,dx.$$

But this slab contains $N\,dx$ electrons, so that the work (W) done per second on each electron is given by

$$W = 2\kappa P/N,$$

and substitution from equation (2) gives us

$$W = (4\pi e^2/\epsilon_0 mc)\,(\nu/p^2)\,P. \tag{3}$$

This work increases the gas-kinetic energy of the electron from the value U, which is in equilibrium with the gas molecules, to the value $U+\Delta U$. It is known from theory and from laboratory experiments that the average energy lost per collision by an electron under these conditions is proportional to its *excess* energy, and can be represented by $G\Delta U$ where G is a constant which has been estimated by Townsend and his school from experiments on the diffusion of slow electrons in gases (Townsend 1925). When equilibrium conditions have been set up under the application of a steady wave we can therefore express the fact that the work done on each electron per second is equal to the energy lost in the ν collisions made per second by writing

$$W = \nu G\Delta U. \tag{4}$$

We now wish to express energies in terms of collisional frequencies, so we relate the electron's velocity u to its collisional frequency ν and its mean free path l by the expression $u = \nu l$. We then have $U = \frac{1}{2}mu^2 = \frac{1}{2}ml^2\nu^2$ and $\Delta U = ml^2\nu\Delta\nu$ where $\Delta\nu$ represents the increase of ν due to the interacting wave. Substitution in equation (4) leads to

$$\Delta\nu = W/ml^2\nu^2G. \tag{5}$$

We now consider the passage of the wanted wave through the region which has been 'heated up' by the interacting wave. At each level in the ionosphere the absorption coefficient of the wanted wave is given by equation (2) and, if we assume that all the important absorption occurs near a fixed level, so that ν is roughly constant over the absorbing region, we may write, for the received amplitude of the wanted wave,

$$F = F_0 e^{-\rho} = F_0 e^{-\alpha\nu}, \tag{6}$$

where ρ represents the attenuation of the wave measured in 'nepers' and $\alpha\nu = \rho$. If now the presence of an interacting wave increases the frequency of collision from ν to $\nu+\Delta\nu$, the received amplitude is reduced to the value

$$F' = F_0 e^{-\alpha(\nu+\Delta\nu)},$$

and, if $\alpha\Delta\nu \ll 1$ we may expand the factor $e^{-\alpha\Delta\nu}$ and obtain

$$F' = F_0 e^{-\alpha\nu}(1-\alpha\Delta\nu) = F(1-\alpha\Delta\nu).$$

If ΔF represents the decrease in the amplitude, F, of the wanted wave we therefore have

$$\Delta F/F = \alpha\Delta\nu, \tag{7}$$

or, after substitution from equations (3) and (5),

$$T = \Delta F/F = \alpha(4\pi e^2/\epsilon_0 cm^2)\,(P/p^2)\,(l^2G\nu)^{-1}. \tag{8}$$

We shall call the quantity $\Delta F/F$ the 'coefficient of transferred absorption' and shall write it as T. It represents the magnitude of the interaction effect at zero frequency. It is shown in appendix A that the coefficient (m) of modulation transferred from the interacting to the wanted wave at modulation frequency f is given by

$$m = T\{2\mu/(1+\tfrac{1}{2}\mu^2)\}\{1+4\pi^2f^2\tau^2\}^{-\frac{1}{2}}, \tag{20a}$$

where μ is the coefficient of modulation of the interacting wave and τ is the relaxation time of the ionospheric electrons.

It will sometimes be more convenient to substitute for α from equation (6) and write the fractional decrease of amplitude in the form

$$T = \Delta F/F \doteq (\rho/\nu)(4\pi e^2/\epsilon_0 cm^2)(P/p^2)(l^2G\nu)^{-1}. \tag{9}$$

Another useful form of this expression containing the electron's velocity u is obtained by using the relation $u = \nu l$ and we then obtain

$$T = \Delta F/F = \rho(4\pi e^2/\epsilon_0 cm^2)(P/p^2)(Gu^2)^{-1}, \tag{10}$$

or, with $\frac{1}{2}mu^2 = \frac{3}{2}kT$

$$T = \Delta F/F = \rho(4\pi e^2/\epsilon_0 cm)(P/p^2)(3GkT)^{-1}. \tag{11}$$

Equation (7) may be rewritten

$$\Delta F/F = \alpha\Delta\nu = \rho\Delta\nu/\nu.$$

Measurement shows that when the interacting wave has a frequency of 200 kcyc./sec. and comes from a 100 kW sender at a distance of 150 km. $\Delta F/F$ is of the order of 0·05 and with $\rho \doteq 3$ we find $\Delta\nu/\nu$ is of the order 0·01. Hence the incidence of such a wave on the ionosphere increases the collisional frequency by about 1 %. It follows that the effective temperature of the electrons increases by about 2 % when this wave is incident.

(b) *Effect of earth's magnetic field*

To include the effect of the earth's magnetic field in the calculations is, in general, complicated, but the salient points can be seen simply in terms of Appleton's (1932) well-known magneto-ionic theory. The most important modification is to the absorption coefficient of the interacting wave as given by equation (2). This takes two forms according as the 'quasi-longitudinal' or the 'quasi-transverse' approximation (Booker 1935) is made, and for each approximation we have to consider separately the absorption of the 'ordinary' and the 'extraordinary' wave of the magneto-ionic theory. The results of this consideration are shown in table 5, which lists factors which replace p^{-2} in equation (2) when the magnetic field is included.

We first note that, for all directions of transmission, the extraordinary wave is strongly absorbed near the gyro-frequency ($p = p_L$ or p_H) so that we should expect to find a marked increase in the magnitude of wave-interaction when the frequency of the interacting wave approaches this value. Bailey (1937 *a*, *b* and *c*) has previously

discussed this phenomenon theoretically and observed it experimentally, and has named it 'gyro-interaction'. We are not further concerned with gyro-interaction in this paper.

We next note, from table 5, that as the frequency p of the interacting wave is decreased, the factor increases until, for cases (*b*), (*c*) and (*e*), p is somewhat less than the gyro-frequency, and then tends to a limiting value determined by the magnitude of p_L or p_T, or, roughly, the magnitude of p_H. In the case, however, of the ordinary wave travelling in a direction which satisfies the quasi-transverse approximations (case (*d*)), the factor, and therefore the magnitude of the wave-interaction, continues to increase as the frequency is decreased. We should therefore expect to find the magnitude of the interaction produced by a low-frequency wave to depend on the direction of propagation of that wave.

Table 5. Factor to replace the factor p^{-2} in equation (2)

approximation	factor
(*a*) no steady magnetic field	p^{-2}
(*b*) quasi-longitudinal—ordinary wave	$(p+p_L)^{-2}$
(*c*) quasi-longitudinal—extraordinary wave	$(p-p_L)^{-2}$
(*d*) quasi-transverse—ordinary wave	p^{-2}
(*e*) quasi-transverse—extraordinary wave	complicated expressions which can be shown to give a maximum absorption at the gyro-frequency, and for low frequencies to tend to p_T^{-2}

The results of this theoretical section as summarized in equations (8), (9), (10) and (11) which give the coefficient of transferred absorption in the absence of a magnetic field, and table 5, which gives the effect of introducing a magnetic field, will be used in the next section, to discuss the results of the measurements described in § 5.

7. Discussion of results

(*a*) *The determination of ν from $G\nu$*

We have previously given, in tables 2 and 3, the heights of the regions of interaction, and the measured values of the electronic relaxation time τ in these regions. It is shown in appendix A, equation (21), that $\tau = 1/G\nu$ so that as a result of the experiments the value of $G\nu$ is known at a known height, and if the value of G can be assumed ν can be determined. The best value of G, deduced from measurements in the laboratory, is taken by Bailey (1937*a*) to be $2{\cdot}6 \times 10^{-3}$, and this, with the results of tables 2 and 3, gives values for ν ranging from $2{\cdot}1 \times 10^5$ to 11×10^5 with a mean value 5×10^5 sec.$^{-1}$. The measured heights of the interaction vary from 75 to 97 km. with a mean of 86·4 km. It is clear from the tables that there is considerable variation in the measured heights and relaxation times, and some of this variation

is undoubtedly due to experimental errors. A phase experiment of the type discussed in §4 takes about 20 min. with the present technique, and since the complications discussed in appendix B undoubtedly occur we do not feel justified in drawing more than general conclusions from the results.

The broad conclusion, that ν has the value 5×10^5 sec.$^{-1}$ at a level of about 85 km. is in good accord with other data. In particular it agrees with that previously found by Bailey (1935) who based his results on the measurements which van der Pol & van der Mark (1935) made on the amplitude of the transferred modulation.

(*b*) *Comparison of theoretical and experimental values of the coefficients of transferred absorption*

The values of the 'equivalent coefficients of transferred absorption' calculated from equation (11), with suitable substitutions from table 5, are shown in table 6 together with the observed values extracted from table 4. The value taken for ρ in the calculations is derived from observations of the pulses emitted by the wanted station.

TABLE 6. EQUIVALENT COEFFICIENTS OF TRANSFERRED ABSORPTION FOR 100 kW SPHERICALLY RADIATED BY AN INTERACTING STATION DISTANT 150 KM. FROM THE POINT OF INTERACTION

Calculations are from equation (11) and table 5, with the following values inserted:

$\rho = 3$, corresponding to a reflexion coefficient of 0·05 for the wanted wave at oblique incidence.

$(2\pi e^2/\epsilon_0 cm) = 5{\cdot}3 \times 10^{-2}$ cm.^{3} sec.$^{-2}$.

$P = 3{\cdot}5 \times 10^{-4}$ ergs cm.$^{-2}$ sec.$^{-1}$ for 100 kW spherically radiated to a distance of 150 km.

$G = 2{\cdot}6 \times 10^{-3}$.

$T = 200°$ K.

$k = 1{\cdot}37 \times 10^{-16}$ erg deg.$^{-1}$.

	calculated			
		ordinary		
frequency of interacting wave (kcyc./sec.)	extraordinary quasi-longitudinal and approx. quasi-transverse	quasi-longitudinal	quasi-transverse	observed values from table 4 (serial number of experiment in brackets)
1013	$1{\cdot}4 \times 10^{-1}$	$2{\cdot}4 \times 10^{-3}$	$1{\cdot}3 \times 10^{-2}$	$2{\cdot}5 \times 10^{-2}$ (4, 7)
200	$1{\cdot}1 \times 10^{-2}$	$5{\cdot}8 \times 10^{-3}$	$3{\cdot}3 \times 10^{-1}$	$2{\cdot}0 \times 10^{-2}$ (1, 3, 6, 8, 9, 10)
90·2	$8{\cdot}7 \times 10^{-3}$	$6{\cdot}6 \times 10^{-3}$	1·6	$< 5 \times 10^{-3}$ (2)
68	$8{\cdot}4 \times 10^{-3}$	$6{\cdot}9 \times 10^{-3}$	2·8	$< 2 \times 10^{-3}$ (5)

In calculating from equation (11) we have assumed a knowledge of G and of the ionospheric temperature; we may, however, calculate from equation (9) and use the measured value of $G\nu$ so as to avoid any assumption about the value of G. We then have to assume reasonable values for ν and for the mean free path l and we obtain a result of the same order, as is indeed obvious if we remember that the measured value of $G\nu$ agrees with accepted values of G and ν.

In attempting to relate the observed and experimental results we first recall that, in the theoretical calculations of table 6, we have assumed that the interacting wave arrives at the region of interaction with its power undiminished; if it suffers appreciable absorption before it gets there the interaction will be less than that calculated.

We first consider the tabulated values of the coefficient of transferred absorption when the frequency of the interacting wave is 200 kcyc./sec. since this represents the case which we have studied in most detail in our experiments. We find that the calculated values lie on each side of the observed value and differ from it by a factor of less than 10. In view of the uncertainty concerning the precise magnitude of G this agreement seems satisfactory and we do not consider it profitable, at this stage, to consider further the absolute magnitude of the coefficient. We do, however, attach considerable significance to the relative magnitudes of the coefficient determined with interacting waves of different frequency, and shall examine these in more detail.

It is first interesting to note that the coefficient measured for experiment (1) of table 4 was about twice that measured in experiments (3), (6), (8), (9) and (10). This difference is probably attributable to the fact that, at the interaction point operative in the first experiment, the interacting wave was travelling more nearly transverse to the magnetic field, and its polarization corresponded to that of the 'ordinary' wave of the magneto-ionic theory.

We next examine the figures for the two low-frequency interacting waves and note that the observed coefficient of transferred modulation is less than 1/10 of that observed with an interacting wave of frequency 200 kcyc./sec. We can at once reject the possibility of interaction produced by an ordinary wave propagated with quasi-transverse direction since this would lead to an excessive amount of interaction. We next note that the calculated magnitudes of the interaction on this frequency for the other directions of transmission are very little different from those calculated for 200 kcyc./sec. We are driven to explain the much smaller observed interaction on the lower frequencies by the assumption that the full power of the interacting wave does not reach the region where the wanted wave is absorbed, either because the interacting wave is reflected or is considerably absorbed below this level.

In contrast to the case just considered it is next of special interest to inquire what interaction occurs between two waves on nearly the same frequency, which would be expected to be absorbed nearly at the same level in the ionosphere. Experiments numbers 4, 9 and 10 of table 4 are of this type, and the results listed in table 5 opposite the frequencies 1013 and 200 kcyc./sec. summarize the observations. For the frequency 1013 kcyc./sec. it is seen that the observed value is in good agreement with the theoretical values deduced both for the extraordinary wave and for the ordinary wave on the quasi-transverse assumption. On a frequency of 200 kcyc./sec. the possible theoretical values embrace a wider range but even so there is no disagreement between theory and experiment.

(*c*) *The possibility of self-interaction*

In the foregoing sub-section it has been emphasized that the maximum amount of interaction can only take place if the interacting and the wanted waves are absorbed in the same place, and the small interaction when the two wave-lengths have been different has been ascribed to the fact that the regions of absorption do not completely overlap. It is therefore of interest to inquire what would happen if the two wave-lengths were exactly the same and if they were travelling in the same direction so that the two absorption coefficients were identical at each point. The wanted and the interacting wave would then be one and the same, and the phenomenon could be called self-interaction. We believe that useful information could be obtained by measurements of self-interaction.

(*d*) *Effects at sunrise*

The coefficient of transferred absorption as given by equations (8), (9), (10) and (11), is dependent on the electron density N through the quantity ρ, which is the absorption coefficient of the wanted wave, and is proportional to N; so that we might expect the coefficient of transferred absorption to increase as N increases with the approach of day, and as the absorption of the wanted wave increases. Grosskopf (1938) looked for an effect of this kind but did not observe it. In § 5 (*d*) we have explained how we observed precisely the opposite effect, as illustrated in figure 13, and it is now necessary to consider how this could happen.

We first notice that, although the increase of electron density in a certain neighbourhood increases the total energy absorbed from the interacting wave it does not alter the work done on each electron by that wave (see equation (3)). So far as this effect goes, therefore, the coefficient of transferred absorption is not altered by an increase of electron density. If, however, the interacting wave is separately absorbed at some other level before it reaches the level where the interaction occurs then, of course, the power P in equation (3) is decreased, and, with it, the coefficient of transferred absorption. Separate absorption of the interacting wave at some other level *after* it has produced interaction does not alter the interaction in any way.

Turning now to consider the wanted wave, we have already noticed that an increase in its absorption in the region of interaction will increase the coefficient of transferred absorption. An increase of its absorption outside that region is simply equivalent to a decrease in the power emitted by the wanted sender and has no effect on the coefficient.

There are therefore two separate tendencies, acting in opposite senses:

(*a*) Increased absorption of the wanted wave in the region where interaction occurs leads to an increased coefficient of transferred absorption.

(*b*) Increased absorption of the interacting wave before it reaches the region of interaction leads to a decrease in the coefficient of transferred absorption.

Other changes of absorption do not alter the coefficient of transferred absorption.

We must conclude that, during the period of sunrise, phenomenon (*b*) more than compensates for phenomenon (*a*). A detailed discussion of the possibilities in the light of theories of ionospheric absorption should provide useful information about changes in the distribution of electron density during the period of sunrise.

8. Conclusions and future experiments

We have shown that, if the magnitude of the quantity G is assumed, measurements of the phase of the transferred modulation lead to a knowledge of the frequency ν of collision between electrons and neutral molecules at a height which can be determined. Our measurements have shown that the value of ν is about 5×10^5 sec.$^{-1}$ at a height of 85 km. They also indicate that this is the height at which the majority of the absorption occurs at night, and thereby confirm the suggestion that the main absorption does not occur at the top of the trajectory. In this respect our results differ from those of Bailey & Martyn (1934).

Measurements of the absolute magnitude of the transferred modulation during the night showed reasonable agreement with theory in the cases where the wanted and interacting waves were near the same frequency, e.g. experiments with serial numbers 4 (1 Mcyc./sec.) and 9 and 10 (200 kcyc./sec.) (table 4). Since we expect the absorption of two waves of the same frequency to occur near the same level we take the agreement as providing additional evidence for the correctness of the assumed value for G and the value deduced for ν.

We also find reasonable agreement with theory when one wave has a frequency about 1 Mcyc./sec. and the other about 200 kcyc./sec. (experiments 1, 3, 6, 7 and 8). This leads us to believe that the waves of these two frequencies are absorbed near the same levels during the night.

With interacting waves of frequencies 90 and 68 kcyc./sec. the interaction is considerably less than would be expected and it appears that these waves must be reflected from heights less than 85 km., the level of absorption of the waves of higher frequency.

We deduce from the decrease of interaction observed when dawn approaches that the levels of absorption for 200 kcyc./sec. and 1 Mcyc./sec. separate considerably, although they had been roughly coincident during the night. This is presumably due to the establishment during the dawn period of a less sharp gradient of ionization in the region of 85 km.

The measuring apparatus used in these experiments has intentionally been kept simple, and although well suited for establishing the main principles of the methods, suffered from the disadvantage that it required about half an hour to make a complete phase experiment. During this time ionospheric conditions may have changed so that no great accuracy is claimed for the values of $G\nu$ or of height given in tables 2 and 3. We intend to modify the apparatus so that measurements can be made while the modulation frequency is changed quickly.

In addition to those organizations whose help has been acknowledged in § 2, we wish to express our thanks to several others for help they have given. We wish to thank the Department of Scientific and Industrial Research which has supported the work with a generous grant. One of us (I. J. S.) is indebted to the Government of New Zealand for a scholarship which has enabled him to work in the Cavendish Laboratory on this subject. We are specially indebted to Mr S. W. H. W. Falloon for considerable help with the apparatus and with the observations, to Mr K. Weekes and Mr J. W. Findlay for help in the observations of the pulse transmissions, and to Dr L. G. H. Huxley for allowing us to see the results of some observations which he made at Birmingham on the transmissions described in this paper.

Appendix A

The magnitude of wave-interaction as a function of modulation frequency

In this appendix we extend the discussion of § 6 to include the case of a modulated interacting wave. The analysis was given in the original paper of Bailey & Martyn (1934*b*), we merely repeat it here in order to relate it to the nomenclature which we have used.

If there is a temporal variation in the power, $W(t)$, expended on each electron by the interacting wave, then the equation of energy balance (equation (4) of § 6 (*a*)) of the electrons becomes

$$W(t) = \frac{d}{dt}(\Delta U) + \nu G \Delta U, \tag{12}$$

or, if we substitute for ΔU in terms of $\Delta \nu$ as in § 6 (*a*):

$$W(t)/ml^2\nu = \frac{d}{dt}(\Delta\nu) + \nu G\,\Delta\nu. \tag{13}$$

The power, $W(t)$, expended on each electron per second is related to the power-flux P in the interacting wave by the expression

$$W(t) = CP(t),$$

where
$$C = (4\pi e^2/\epsilon_0 mc)\,(\nu/p^2) \tag{14}$$

(cf. equation (3)) and we shall write

$$P(t) = P_0(1 + M\cos\omega t) \tag{15}$$

to represent a modulated wave.

If we now substitute from equations (14) and (15) into (13) and solve for $\Delta\nu$ we obtain

$$\Delta\nu = (CP_0/ml^2\nu^2G)\,[1 + M\{1 + (\omega/G\nu)^2\}^{-\frac{1}{2}}\cos(\omega t - \phi)], \tag{16}$$

where
$$\phi = \tan^{-1}(\omega/G\nu).$$

Substitution of this result into equation (7) together with expression (14) for C gives

$$\Delta F/F = \alpha(4\pi e^2/\epsilon_0 cm^2)\,(P_0/p^2)\,(l^2G\nu)^{-1}\,[1 + M\{1 + (\omega/G\nu)^2\}^{-\frac{1}{2}}\cos(\omega t - \phi)]. \tag{17}$$

If the wave *amplitude* is sinusoidally modulated with a coefficient μ there will be an octave component in the modulation of the *power*, but if this is neglected (as it was in our experiments) and we consider only the fundamental component we have

$$M = 2\mu/(1+\tfrac{1}{2}\mu^2). \tag{18}$$

It should be noted that, although μ is always less than unity, M may be greater than unity. In our experiments, for example, μ was 0·8 and M was therefore 1·21.

If we combine expression (18) with expression (17) and substitute from equation (8) for the coefficient of transferred absorption T, we find that

$$\Delta F/F = T[1+\{2\mu/(1+\tfrac{1}{2}\mu^2)\}\{1+(\omega/G\nu)^2\}^{-\frac{1}{2}}\cos(\omega t-\phi)]. \tag{19}$$

Equation (19) shows that, if $T \ll 1$ the wanted wave is modulated with a modulation coefficient m given by

$$m = T\{2\mu/(1+\tfrac{1}{2}\mu^2)\}\{1+(\omega/G\nu)^2\}^{-\frac{1}{2}}, \tag{20}$$

and the phase of this transferred modulation lags on that of the interacting modulation by the angle $\phi = \tan^{-1}(\omega/G\nu)$.

The quantity $1/G\nu$ plays an important part in the phenomena discussed in this appendix and it is of interest to give it a physical meaning. Reference to equation (12) shows that, if the interacting wave is suddenly removed the electrons will lose their excess energy ΔU as given by

$$\frac{d}{dt}(\Delta U)+\nu G\Delta U = 0,$$

or

$$\left.\begin{aligned}\Delta U &= (\Delta U)_0\, e^{-\nu G t}\\ &= (\Delta U)_0\, e^{-t/\tau},\end{aligned}\right\} \tag{21}$$

where $\tau = 1/G\nu$ can be called the 'relaxation time' of the electrons. We shall often use equation (20) written as

$$m = T\{2\mu/(1+\tfrac{1}{2}\mu^2)\}\{1+4\pi^2 f^2\tau^2\}^{-\frac{1}{2}}. \tag{20a}$$

Appendix B

Effects of transmission over multiple paths

In this appendix we consider how the following complicating processes will modify the amplitude and phase of the transferred modulation measured at the receiver:

(*a*) Two ionospheric waves reach the receiver by different paths and each has modulation transferred to it at some point along its trajectory. We shall restrict ourselves to the case where one of the waves is much weaker than the other.

(*b*) A single ionospheric wave has modulation transferred to it at two different points along its trajectory. We shall restrict ourselves to the case where one modulation is much weaker than the other.

(*c*) A single ionospheric wave with modulation transferred to it at a single point in its trajectory is received simultaneously with a ground wave which carries no transferred modulation.

We first consider case (*a*). Let us suppose that, as indicated in figure 14, downcoming waves from W are received at R simultaneously by way of reflexion from region E at the point E and from region F at the point F, and that interaction takes place on the two waves separately at the points B and C. Let the path differences for each wave separately be given by

$$ICFR - IAR = d_2, \quad IBER - IAR = d_1,$$

and let the amplitude of the two radio-frequency waves be A_1 and A_2 on arrival at R and their coefficients of transferred modulation be m_1 and m_2. If then the angular

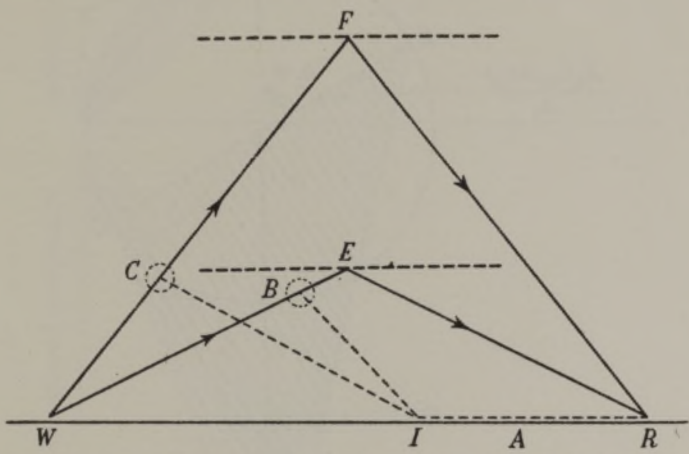

FIGURE 14. Schematic diagram to illustrate discussion of simultaneous reception of two downcoming waves each carrying transferred modulation.

radio-frequency is denoted by p and the angular audio-frequency by ω, the two waves produce e.m.f.s at the receiver given by

$$\left.\begin{aligned} &A_1[1+m_1\cos\{\omega t-\omega d_1/c-\phi(\omega)\}]\cos pt,\\ \text{and}\qquad &A_2[1+m_2\cos\{\omega t-\omega d_2/c-\phi(\omega)\}]\cos(pt+\psi).\end{aligned}\right\} \tag{22}$$

In these expressions the phase of the modulation is referred to a zero which is the phase of the modulation on the wave received direct from I by the path IAR; the phase angle $\phi(\omega)$ is given by $\tan\phi = \omega\tau$ and represents the phase-lag due to the effect of the electronic relaxation time in the ionosphere; and the phase angle ψ is included to represent the difference in the phase of the radio-frequency e.m.f.'s received by the two paths.

We first rewrite expressions (22) as

$$\left.\begin{aligned} &A_1(1+m_1\cos\theta_1)\cos pt,\\ &A_2(1+m_2\cos\theta_2)\cos(pt+\psi),\end{aligned}\right\} \quad \text{where} \quad \left\{\begin{aligned} \theta_1 &= \omega t-\omega d_1/c-\phi(\omega),\\ \theta_2 &= \omega t-\omega d_2/c-\phi(\omega),\end{aligned}\right\} \tag{23}$$

and we then find for the resultant e.m.f. the expression

$$\left\{ \begin{aligned} A_1^2(1+m_1\cos\theta_1)^2 + A_2^2(1+m_2\cos\theta_2)^2 \\ +2A_1A_2(1+m_1\cos\theta_1)(1+m_2\cos\theta_2)\cos\psi \end{aligned} \right\}^{\frac{1}{2}} \cos(pt+\chi), \tag{24}$$

where χ is a phase angle depending on the other variables. If we assume m_1 and $m_2 \ll 1$, the amplitude A of this resultant radio-frequency e.m.f. may now be expressed as

$$A = A_0 \left\{ \begin{aligned} 1 + (A_1/A_0)^2 m_1 \cos\theta_1 + (A_2/A_0)^2 m_2 \cos\theta_2 \\ + (A_1A_2/A_0^2)(m_1\cos\theta_1 + m_2\cos\theta_2)\cos\psi, \end{aligned} \right\} \tag{25}$$

where $$A_0^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\psi.$$

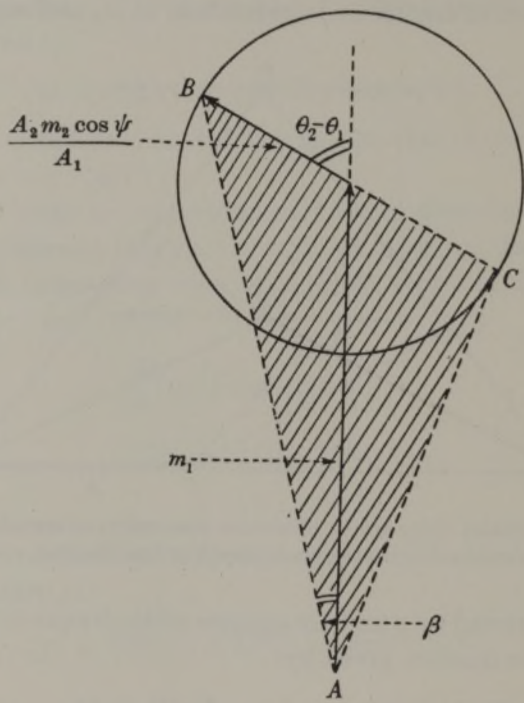

FIGURE 15. To illustrate meaning of equation (25).

The last three terms inside the bracket represent the modulation of the wave which, in general, may be quite complicated. In order to simplify matters we shall assume that A_2 is small compared with A_1, so that the expression in the bracket reduces approximately to

$$1 + m_1\cos\theta_1 + (A_2/A_1)\, m_2 \cos\psi \cos\theta_2. \tag{26}$$

The modulation then consists of two superimposed sinusoidal modulations with amplitudes m_1 and $(A_2/A_1)\, m_2 \cos\psi$ and with a phase difference given by

$$\theta_1 - \theta_2 = \omega(d_2 - d_1)/c.$$

The resultant of these two simultaneous modulations is most simply visualized by a vector diagram such as that of figure 15 in which the vectors are supposed to rotate with the frequency of the modulation. It is then clear that the phase of the resultant modulation will differ by the angle β from the phase of the modulation

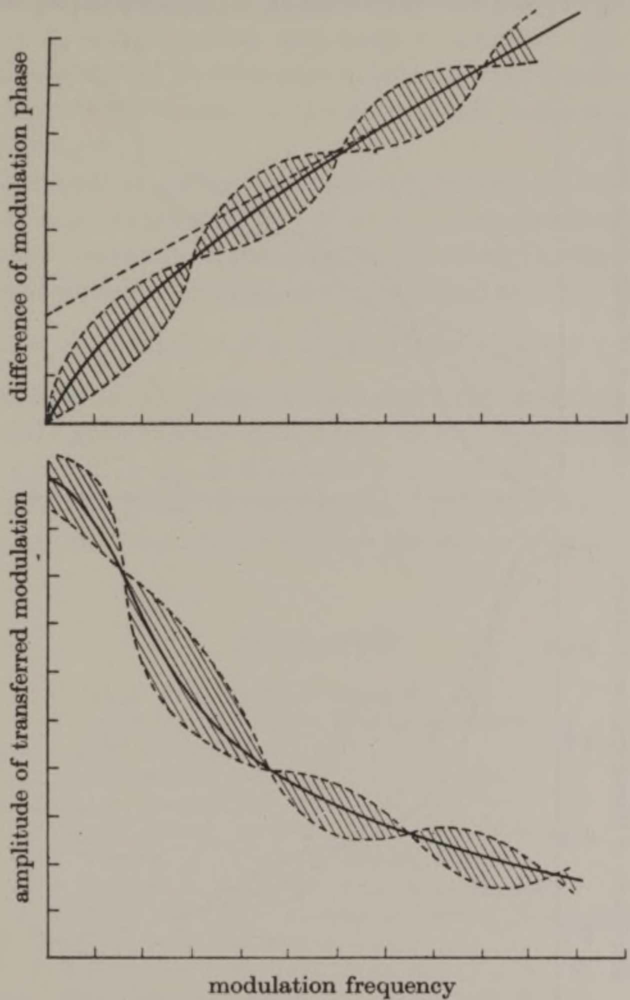

FIGURE 16. The expected effect of interaction on two simultaneously-received downcoming waves. The solid line represents the theory for interaction at one point on a single downcoming wave. For interaction on each of two downcoming waves the resulting curves lie somewhere inside the shaded regions, the precise position depending upon the radio-frequency phases of the two waves.

due to A_1 alone. The magnitude of this phase difference depends on the quantity $A_2 m_2 \cos\psi / A_1 m_1$ and on the phase difference $(\theta_1 - \theta_2)$ between the modulations received separately by the two paths. Small changes in the ionosphere will cause ψ to assume a series of random values and will lead to changes in A_1 and A_2; as these changes occur the amplitude and phase of the received modulation will assume

values represented by vectors in the shaded triangle ABC. In any experiments, therefore, there will be an element of random 'scatter' amongst a set of points plotted to represent the phase or amplitude of the modulation. If the modulation frequency is altered as the experiments proceed, the magnitude of $\theta_1-\theta_2=\omega(d_2-d_1)/c$ will alter, and when $\theta_2-\theta_1=0$ the variations of $A_2m_2\cos\psi/A_1m_1$ will not affect the

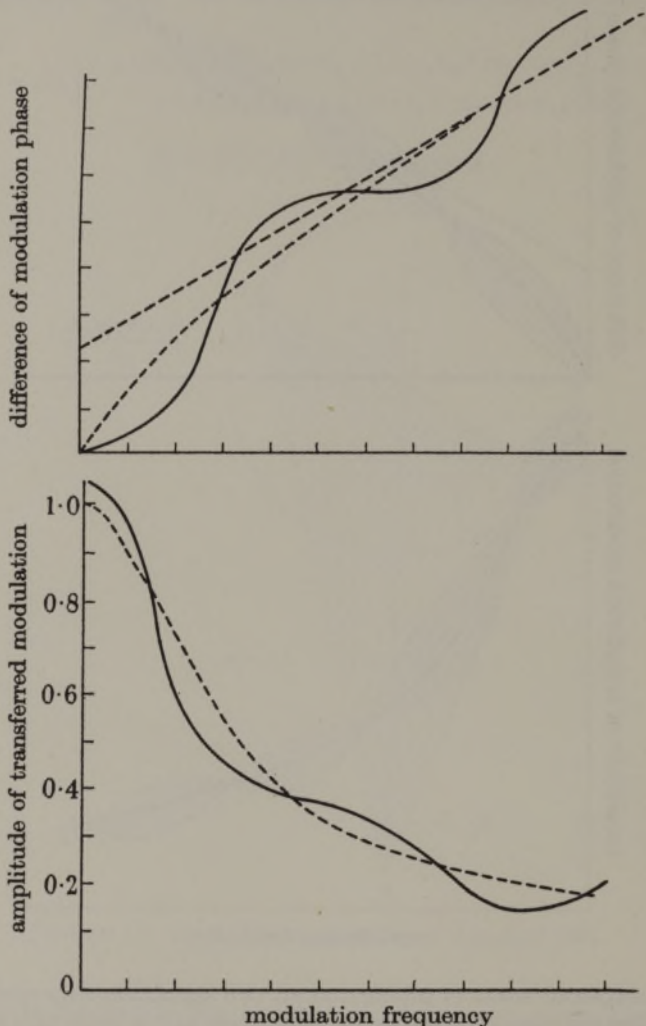

FIGURE 17. The expected effect of simultaneous interaction at two points on a single down-coming wave. The dotted lines represent the theory for interaction at one point, and the solid lines the results for interaction at two points.

phase of the resultant modulation but will have maximum effect on the amplitude, whereas when $\theta_2-\theta_1=\frac{1}{2}\pi$ there will be a marked effect on the phase, but little on the amplitude. If interaction occurred, in the ideally simple way, at one point on one ray, the theories which we have previously given would lead us to expect the plots of experimental results to fall on curves typified by the solid lines of figure 16.

The occurrence of interaction on two transmission paths, between which there was a variable phase, would cause the plots of experimental results to be scattered about these lines, in such a way as to lie always in the regions shown shaded.

To consider case (*b*), where a single ionospheric wave has modulation transferred to it at two different points along its trajectory, we use precisely the same analysis as before and put $A_1 = A_2$, $m_1 \gg m_2$, and keep ψ constant. If, now, A_2/A_1 also remained constant we should find our experimental points lying on a curve such as that of figure 17. There may be some scatter around this curve as a result of changes in the magnitude of A_2/A_1.

We now consider case (*c*), where a single ionospheric wave, with modulation transferred to it at a single point in its trajectory, is received simultaneously with a ground wave which carries no transferred modulation. Our previous expressions represent this case if we take $A_1(=)A_2$ and $m_2 = 0$, and equation (25) then becomes

$$A = A_0[1 + \{(A_1/A_0)^2 + (A_1A_2/A_0^2)\cos\psi\}m_1\cos\theta_1].$$

This expression shows that the phase of the transferred modulation is not affected by the presence of the ground wave except that *it may be completely reversed* if $\cos\psi$ is negative and $|A_2\cos\psi| > A_1$. It is important to remember this possibility when interpreting the absolute phase of the transferred modulation in the presence of a relatively strong ground wave received from the wanted transmitter.

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